WORKSHOP BOOKLET

Hopf Algebroids and Noncommutative Geometry

School of Mathematica Sciences
Queen Mary University of London
327 Mile End Rd, London E1 4NS

July 12-14, 2023
Website: https://hopfalgebroid.wordpress.com/

1. Practical Information

Location: Lectures take place from 9am in the Maths Seminar Room 503 on the top floor of the maths building. This building is located on Mile End Rd on the corner of the Queen Mary Campus nearest to Stepney Green metro station. Please see the map below.

There is also a maths common room 504 next to the Maths Seminar Room with a credit-card operated coffee machine. **Tea/Coffee breaks 10-10.30am & 3.30-4pm** will take place here.

Building access: Doors to the main building should be open during conference hours but otherwise will need card access to enter. Exit is possible during most hours by pressing the green exit button (outside of which there are instructions to call security.)

Access to the maths common room requires a staff card. During coffee breaks someone will be around to let anyone in, please knock to draw attention if you end up on the wrong side of this door.

Talks: Morning talks are 1 hour, afternoon talks are generally 45 minutes or 30 minutes. Please check the latest schedule on the website or latest version of this booklet.

For remote participants, a zoom link is provided on the conference website.

Meals: The workshop is not able to offer subsistence but there is a subsidised campus cafeteria **The Curve** (see map) where a reasonable cooked meal can be had for about £6. It is open for **breakfast 8-10.30am, lunch 11.30am-2.30pm and dinner 4-7pm**. There are also plenty of restaurants in the local area and environs, notably Brick Lane is about 15 minutes walk.

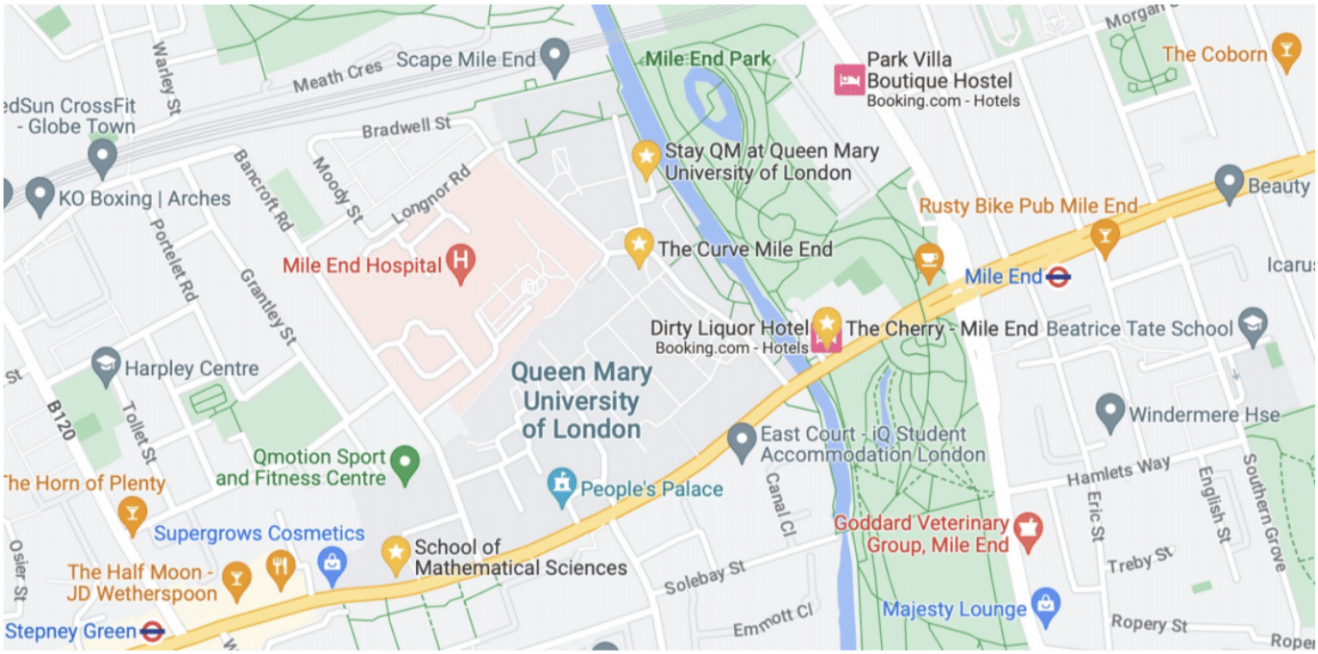
Conference Dinner: Thursday 13th July 6.30pm at **Coburn Arms** (see map). The conference will cover the main costs (communal starters, main and a drink) for all registered participants, not just speakers, with anything else available to order at your own cost. But please let us know as below.

Registration: There is no registration fee for the conference but all participants were asked to register by email with Xiao Han. Based on this information, badges will be available for collection in the maths common room during the first coffee break and thereafter from Xiao.

Registration for the conference dinner: please confirm your attendance by email to Xiao Han as soon as possible and at latest by **2pm Wed 12th** at the latest so that we can give notice to the venue. Also please inform us if there are any particular dietary restrictions to pass on to the venue.

Accommodation: We were able to support a few participants with accommodation in France House on Campus ('Stay QM' on the map) and at The Cherry (see map).

Contacts: x.h.han@qmul.ac.uk (07902 087178) & s.majid@qmul.ac.uk (07910 521069)



2. Schedule (please check online for any updates)

* denotes zoom talks.

Times in London	Wednesday 12 July	Thursday 13 July	Friday 14 July
9.00-10.00	Kowalzig	Xu	Schauenburg
10.00-10.30	**Coffee break**	**Coffee break**	**Coffee break**
10.30-11.30	Vercruysse*	Kraehmer	Brzezinski*
11.30-12.30	Meyer*	Saracco	Kaufmann
12.30-14.00	**Lunch break**	**Lunch break**	**End of workshop**
14.00-14.45	Han	Gavarini	
14.45-15.30	Weber*	Landi*	
15.30-16.00	**Coffee break**	**Coffee break**	
	16.00-16.30 Chemla	**16.00-16.45** Laugwitz*	
	16.30-17.00 McCormack		
	17.00-17.30 Zanchettin	**16.45-17.30** Morales Parra*	
18.30-21.00		**Conference dinner**	

3. Abstracts (in alphabetical order)

Tomasz Brzezinski (Swansea University) Hopf heaps and Hopf algebras of translations

> Grunspan and Schauenburg have proven that quantum torsors (introduced by the former) are closely connected with Hopf-Galois objects. In this talk I will present in part the dualisation of this connection and in part a new characterisation of quantum cotorsors or Hopf heaps in terms of Galois-type actions of the associated translation Hopf algebras (not available in the dual setup). The talk is based on a joint work with Magorzata Hryniewicka.

Sophie Chemla (Pierre and Marie Curie University) Left Hopf algebroids and (quasi-) Frobenius extensions

> We study when left (op)Hopf algebroids in the sense of Schauenburg give rise to a Frobenius or quasi-Frobenius extension. The case of Hopf algebroids in the sense of Böhm was treated by G. Böhm. Contrary to Hopf algebroids, (op)Hopf left algebroids don't necessarily have an antipode but their Hopf-Galois map is invertible. We make use of recent results about left Hopf algebroids (Schauenburg, Kowalzig). Our results are applied to the restricted enveloping algebra of a restricted Lie-Rinehart algebra.

Fabio Gavarini (University of Rome "Tor Vergata") Duality functors for quantum groupoids

> I will present a joint work with Sophie Chemla, concerning suitably defined quantum groupoids and the main duality functors that interchange them. Here "quantum groupoid" stands for a formal deformation of either one of the bialgebroids canonically associated with any Lie-Rinehart algebra - the latter being the algebraic incarnation of a Lie algebroid. The theory was started by Ping Xu, who introduced the first natural type of quantum groupoid. Our first contribution was to introduce the second natural type of quantum groupoid. Second, we developed in some detail the formalism of linear duality, which connects the two types of quantum groupoids via a category antiequivalence. Third, we devised a suitable version of "quantum duality principle" for quantum groupoids: this extends the one for quantum groups (due to Drinfeld), providing explicit equivalences between the two types of quantum groupoids. In the talk I will present a swift overview of the above mentioned constructions, keeping as a guideline the familiar case of quantum groups, while pointing out the key differences that arise with quantum groupoids.

Xiao Han (Queen Mary University of London) Bisections and cocycles on Hopf algebroids and coquasi-bialgebroids

> We introduce left and right groups of bisections of a Hopf algebroid and show that they form a group crossed homomorphism with the group of bialgebroid automorphisms. We also introduce a nonAbelian cohomology governing cotwisting of a Hopf algebroid with base. For the Ehresmann-Schauenburg Hopf algebroid of a quantum principal bundle or Hopf-Galois extension, we show that the group of bisections reduces to the group of bundle automorphisms, and give a description of the nonAbelian cohomology in concrete terms in two cases: the bundle space subject to a 'braided' commutativity condition and the bundle space is a cleft extension or 'trivial' bundle. Next we show that the action bialgebroid associated to a braided-commutative algebra in the category of crossed (or Drinfeld-Yetter) modules over a Hopf algebrais an fact a Hopf algebroid. We show that the bisection groups are again isomorphic and can be described concretely as a natural space of multiplicative cocycles. Finally, we introduce

coquasi-bialegbroid and their gauge equivalence, then give an concrete example on Coquasi-Connes-Moscovici's bialgebroid, which based on a measuring.

Ralph Kaufmann (Purdue University) From Feynman categories to bi- and Hopf algebroids.

The morphisms of Feynman categories, which are special monoidal categories, are the basis for a bialgebra structure. The coproduct is deconcatenation and the product is given by the monoidal product. The bi-algebra equation is satisfied due to the special properties of a Feynman category. The categorical origin yields source and target maps. There is a Takeuchi-Quillen type of filtration and the obstruction to an anti-pode is given by grouplike elements. Inverting these and making them commutative gives a quantum type of deformation.

Niels Kowalzig (University of Rome "Tor Vergata") Brackets and products from centres in extension categories

Building on Retakh's approach to Ext groups through categories of extensions, Schwede reobtained the well-known Gerstenhaber algebra structure on Ext groups over bimodules of associative algebras both from splicing extensions (leading to the cup product) and from a suitable loop in the categories of extensions (leading to the Lie bracket). We show how Schwede's construction admits a vast generalisation to general monoidal categories with coefficients of the Ext groups taken in (weak) left and right monoidal (or Drinfel'd) centres. In case of the category of left modules over bialgebroids and coefficients given by commuting pairs of braided (co)commutative (co)monoids in these categorical centres, we provide an explicit description of the algebraic structure obtained this way, and a complete proof that this leads to a Gerstenhaber algebra is then obtained from an operadic approach. Conjecturally, the algebraic structure we describe should produce a Gerstenhaber algebra for an arbitrary monoidal category enriched over abelian groups.

Ulrich Kraehmer (Technische Universitaet Dresden) Differential operators on the cusp

In characteristic 0, the ring of differential operators over an affine variety is well-known to be a Hopf algebroid if the variety is smooth, as it is in this case the universal enveloping algebra of the Lie-Rinehart algebra of vector fields. For a non-smooth variety the latter is (conjecturally) not the case. In this talk I will report on recent work joint with Myriam Mahaman in which we show that for the cusp, the differential operators nevertheless are a Hopf algebroid.

Giovanni Landi (University of Trieste) Hopf algebroids, Atiyah sequences and noncommutative gauge theories

We consider noncommutative principal bundles which are equivariant for a triangular Hopf algebra and analyze an associated (noncommutative) gauge groupoid as well as an Atiyah sequence of braided infinite dimensional Lie algebras which are related to gauge transformations acting on connections. From this sequence we derive a Chern-Weil homomorphism and braided Chern-Simons terms. We present explicit examples over noncommutative spheres.

Robert Laugwitz (University of Nottingham) Braided Hopf crossed modules and groupoids of algebras

Majid defined the concept of a braided Hopf crossed module, based on a braided Hopf algebra in a category of Yetter–Drinfeld modules, as a generalization of a crossed module of groups in [ArXiv:1208.6265]. In the same paper, Majid describes how to obtain a certain type of quantum groupoid from Hopf crossed

modules (which are braided Hopf crossed modules with trivial coaction) via the Majid–Radford biproduct construction. This result generalizes the classical construction of a 2-group (i.e., a groupoid in the monoidal category of groups) from a crossed module of groups. In his PhD thesis, Aguiar introduced the concept of a category object in a regular monoidal category. In this talk, I will propose a definition of a groupoid object in a regular monoidal category building on Aguiar's framework. I will then show how a general braided Hopf crossed module can be used to construct such a groupoid object in the monoidal category of k-algebras via the Majid–Radford biproduct. This gives a possible answer to a question raised in Majid's paper.

Leo Sean McCormack (Queen Mary University of London) Centers and exact sequences of braided Hopf algebras

Viewing braided Hopf algebras as important generalisations of super-groups that incorporate 'braided statistics', we look at the problem of formulating exact sequences of braided Hopf algebras, as a generalisation of exact sequences of ordinary Hopf algebras and the ordinary Hopf centre. We first introduce a notion of braided Hopf algebra centre using of braided Tannaka-Krein reconstruction theory from the braided subcategory of 'commutative comodules' of a braided Hopf algebra. After characterising this braided centre and recovering features seen in the classical case, we propose a tentative definition of an exact sequence for braided Hopf algebras.

Ralf Meyer (University of Goettingen) Algebraic Cuntz-Pimsner algebras as limits of diagrams in the bicategory of rings and bimodules

Many interesting C*-algebras may be defined as Cuntz-Pimsner algebras of a single C*-correspondence or a product system of C*-correspondences. This construction may be interpreted as a (bi)limit in the bicategory of C*-correspondences, provided the C*-correspondences involved are regular. In this lecture, I will speak about a purely algebraic analogue of this, with bimodules over rings replacing C*-correspondences. In this setting, if the bimodules involved are finitely generated projective as right modules, then the (bi)limit exists and is described through Cohn localisation. For a single bimodule with this extra property, the bilimit agrees with the algebraic version of the Cuntz-Pimsner algebra. In particular, for the bimodule describing a regular graph, the bilimit is the Leavitt path algebra of the underlying graph.

Morales Parra, Juan Carlos (Heriot-Watt University) Hopf algebroids and Lie bialgebroids in 3d gravity

As a gauge theory, symmetries of Chern-Simons theory are described by (Poisson)-Lie groups. Reason why quantum groups have been considered as candidates to describe the symmetries of the quantized theory. An encarnation of this situation arises in the setting of 3d gravity, where quantum doubles and kappa-Poincare type groups have been proposed as algebraic structures to capture the symmetries of quantum 3d gravity (and its non-commutative phase spaces). In the talk we will explore how these quantum groups could be enriched with the structure of Hopf-algebroids. This suggests they could play a key role to encode symmetries of quantum topological field theories. Also, we will see how the semi-classical approach via classical r-matrices/Lie bialgebras in 3d gravity could be extended to classical dynamical r-matrices/Lie bialgebroids, establishing in this way a programme regarding the identification of possible Hopf-algebroids representing symmetries of 3d quantum gravity. Based in work in progress under the supervision of Prof. Bernd Schroers.

Paolo Saracco (ULB – Université Libre de Bruxelles) Smash and crossed product decompositions of universal enevloping algebras and Lie-Rinehart algebra connections

> Universal enveloping algebras of Lie-Rinehart algebras, like their classical analogues, allow us to approach questions and results in Lie theory and differential geometry from a (Hopf) algebraic perspective, with all the advantages that associative algebra and representation theory can give. In a recent joint work with X. Bekaert and N. Kowalzig, we showed how decompositions of LieRinehart algebras as (curved) semi-direct products correspond to factorizations of their universal enveloping algebras as (crossed) smash products, proving a number of structure theorems for universal enveloping algebras of Lie-Rinehart algebra extensions. During this talk, we will see how these results follow from a Blattner-Cohen-Montgomery theorem for Hopf algebroids and how they can be used, for instance, to show that any flat (or curved) invariant Ehresmann connection on a principal bundle provides a factorisation of the associative algebra of invariant differential operators on the total space as a smash (or crossed) product of the associative algebras spanned by invariant differential operators which are tangential to the fibres and by differential operators on the base manifold, respectively.

Peter Schauenburg (Université de Bourgogne) Module categories over finite quantum groupoids

> We review an easy way to (re)construct a quantum groupoid over which a given tensor category is the category of moules. Part of the assumptions on the category ensure that it is a category of modules over some algebra (which is not at all a quantum groupoid however). We discuss necessary and sufficient conditions for a suitable (finite) quantum groupoid for the situation to exist, some structural peculiarities of the reconstructed object, and classes of examples.

Joost Vercruysse (ULB – Université Libre de Bruxelles) The Hopf algebroid of partial representations

> Hopf algebroids play the role of groupoids in non-commutative geometry. In this talk I will discuss a class of Hopf algebroids that appear in the study of partial representations of Hopf algebras. Geometrically, partial representations allow to describe partially defined symmetries in non-commutative geometry, just as usual actions of Hopf algebras describe global symmetries. I will explain some of the main features and remaining challenges of this young theory, including some anomalies in the dual theory of partial corepresentations.

Thomas Weber (University of Turin) Infinitesimal braidings and pre-Cartier categories

> Infinitesimal braidings are natural transformations in symmetric categories which can be used to construct braidings on the formal power series expansion of the category. Most prominently, for complex semisimple Lie algebras g this leads to a braiding on formal power series of Ug-modules which is equivalent to that of the corresponding Drinfeld-Jimbo quantum group. In this talk we propose a more general approach to infinitesimal braidings which applies to arbitrary braided categories. The motivating idea is to understand such an infinitesimal braiding as a first order deformation of a given braiding. We call such categories pre-Cartier, as they generalize previously studied Cartier categories. In case of (co)quasitriangular bialgebra (co)modules we discuss the algebraic structure equivalent to an infinitesimal braiding. These are Hochschild 2-cocycles satisfying a deformed version of the quantum Yang-Baxter equation, while they are Hochschild 2-coboundaries in the Cartier (co)triangular Hopf algebra framework. We discuss several examples of infinitesimal braidings, particularly on

q-deformed GL(2), Sweedler's Hopf algebra and via twisting. As main results we provide an infinitesimal FRT construction and Tannaka-Krein reconstruction theorem for pre-Cartier coquasitriangular bialgebras. The former admits canonical non-trivial solutions and consequently induces non-trivial infinitesimal braidings on all FRT bialgebras. The talk is based on a collaboration with Ardizzoni, Bottegoni and Sciandra.

Ping Xu (Penn state University) Quantum universal enveloping algebroids

The notion of quantum universal enveloping algebroids (QUE algebroids) was introduced as a unification of quantum groups and star-products. The infinitesimal of a QUE algebroid is a Lie bialgeboid. In this talk, we will discuss some open questions regarding quantization of Lie bialgeboids.

Jacopo Zanchettin (SISSA) Morita equivalence for the Ehresmann-Schauenburg Hopf algebroid

In this talk, I will recall the notion of principal bibundle for commutative Hopf algebroids introduced by El Kaoutit and Kowalzig and adapt it to Schauenburg's Hopf algebroids. Eventually, I will show that any such Hopf algebroid admitting a principal bibundle with a Hopf algebra is isomorphic to the Ehresmann-Schauenburg Hopf algebroid associated with a Hopf-Galois extension. This reproduces the classical result that every Lie groupoid is Morita equivalent to a Lie group if and only if it is the gauge groupoid of a principal bundle. In the last part, I will discuss how to get the same result starting from a monoidal equivalence. This talk is part of a joint work with A. Chirvasitu and M. Tobolsky.